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Matrix models for Yang-Mills interactions*

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Abstract: We introduce a $N \times N$ matrix model with $\mathcal{N} = 2$ supersymmetries and show its relation to the topological rigid string and the topological YM_2. This allows to connect these theories directly. Moreover the construction leads to a new insight in the $N \to \infty$ limit. Finally a quantum mechanical matrix theory which may describe DLCQ dynamics of gauge fields is proposed.

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Contents

Recently there is a growing interest in matrix models. The models are interested on its own [1] and can be applied to various physical systems underlying string theory e.g. M-theory [2], supermembrane theory [3] and QCD (see e.g. [4]).

One of the most interesting models is the matrix model of M-theory. As we are going to use some of the results obtained for this model we briefly recall its basic properties. The model is the dimensional reduction of the 10d SUSY YM down to one dimension. It is not Lorentz invariant. This is intrinsically related to the fact that one tries to build interacting theory of many particle system using quantum mechanics. It is believed that the Lorentz invariance is restored in the large-N limit, where N is the dimension of the matrices. The model has many interesting features e.g. it can properly describe scattering of 11d gravitons [5] and M-theory branes [2].

The purpose of this paper is to construct several matrix models and analyze their behaviour for large N, where N is the dimension of the matrices. The hope is that we shall be able to acquire deeper understanding of the $N \to \infty$ limit. The models we are going to deal with posses world-line $\mathcal{N} = 2$ local supersymmetry i.e. contrary to the M-theory, it is not space-time supersymmetry. For time-independent matrix models the supersymmetry is related to a topological symmetry [6]. In the $N \to \infty$ limit the models reveal close resemblance to 2d topological string theories [7, 8] and 2d topological Yang-Mills theories [9] depending on the way one takes the limit. This indicates that the matrix models can be the proper link between both types of the 2d continuous theories. Although all calculations are made for flat 2d target spaces we believe that the results should hold also for arbitrary Riemann surfaces as targets. The reason is that all the theories have the same underlying topological symmetry and the moduli problem.

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We start our construction from the quantum mechanics of spin-1 particles. The theory has $\mathcal{N} = 2$ local SUSY on world-line [10]. We would like to generalize this construction to many body interacting systems i.e. non-abelian gauge particles. In order to achieve this we introduce the matter as hermitean $N \times N$ matrices. We expect that such matter multiplets will provide a convenient representation of multi-particle states and will naturally lead to interaction between those states. The necessary gauge symmetry is imposed on the world-line. Unfortunately the relevant world-line SUGRA is unknown and we are not going to pursue this track. Instead we discuss two other ways which may provide Poincaré invariant theory. First of all one can construct time independent theories which appears to have a topological symmetry. The symmetry is a part of the world-line $\mathcal{N} = 2$ supersymmetry.

An alternative way is to construct a non-relativistic theory with globally well defined world-line time τ for a multi-particle system. This theory may be interpreted as a DLCQ version of a covariant theory for which $\tau \propto x^+$ and x^- is compactified [11]. The advantages of this frame to investigations of many-particle systems are well known [12, 13]. For the DLCQ one hopes to recover Lorentz invariance in the $N \to \infty$ limit. There is also a heuristic need for infinite matrices. Because we want to think about the matrix model as quantum mechanics of gauge particles we should not expect to get gauge invariant states built from a finite number of gluons.

The outline of the paper is the following. In section 1 we shall recall certain facts about $\mathcal{N} = 2$ world-line supergravity coupled to matter and we shall show how it is related to abelian gauge particles. This section has a review character as all the results are well known. It serves as a motivation for the form of the supersymmetry. Section 2 is devoted to (time-independent) topological matrix models and their relation to 2d topological strings and to the 2d topological YM_2 theory of Witten [9]. In section 3 we shortly describe a matrix model for dynamical gauge fields. There the matrices depend on one continuous parameter ("time").

1. Spin 1 ($\mathcal{N} = 2$) particle

Quantum mechanics of free particles of any spin s is well understood [10]. It is given by $\mathcal{N} = 2s$ extended 1d supergravity coupled to matter multiplets. The relevant action for $N = 2$, $(s = 1)$ case is

$$S = \frac{1}{2} \int d\tau \left(\frac{1}{e} \left(\dot{X} - i \left(\chi\overline{\psi} + \overline{\chi}\psi \right) \right)^2 + i\psi D\overline{\psi} + i\overline{\psi} D\psi + eF^2 \right), \qquad (1.1)$$

where $\psi = \frac{1}{\sqrt{2}}(\psi_1 + i\psi_2)$ and $\psi_{1,2}$ are real, $D\psi \equiv (\partial_\tau - if)\psi$. The same convention holds for χ and for ϵ (see below). All fermions without bar have the $\mathrm{U}_f(1)$ charge $T = -1$ while those with bar have $T = +1$. In (1.1) $(e, \chi, \overline{\chi}, f)$ is the gravity multiplet and $(X, \psi, \overline{\psi}, F)$ is the matter multiplet carrying a space-time index.[1] The

[1] Wherever it will be possible this index will be suppressed in order to simplify notation.

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local supersymmetry transformations are

$$\begin{aligned}
\delta X &= i\epsilon\overline{\psi} + \text{h.c.} \\
\delta\psi &= \frac{\epsilon}{e}\left(-\dot{X} + i\left(\chi\overline{\psi} + \overline{\chi}\psi\right) + ieF\right) \\
\delta F &= \frac{\epsilon}{e}\left(D\overline{\psi} + i\chi F + \frac{1}{e}\overline{\chi}\dot{X} + \frac{i}{e}\chi\overline{\chi}\overline{\psi}\right) + \text{h.c.} \\
\delta e &= -2i\epsilon\overline{\chi} + \text{h.c.} \\
\delta\chi &= D\epsilon \\
\delta f &= 0\,.
\end{aligned} \tag{1.2}$$

The theory has the local $\mathrm{U}_f(1)$ symmetry for which f is the gauge field. The constraints on physical states are $Q|\mathrm{phys}\rangle = \overline{Q}|\mathrm{phys}\rangle H|\mathrm{phys}\rangle = T|\mathrm{phys}\rangle = 0$ $(i = 1, 2)$, where

$$Q = \Pi_X\psi\,, \qquad \overline{Q} = \Pi_X\overline{\psi} \tag{1.3}$$

$$H = \Pi_X^2 - F^2 \tag{1.4}$$

$$T = \overline{\psi}\psi \tag{1.5}$$

are the generators of the supersymmetry, the 1d reparameterizations and the $\mathrm{U}_f(1)$, respectively.

The standard quantization procedure gives $\{\psi^m, \overline{\psi}^n\} = \delta^{mn}$, with all the other anti-commutators equal zero. We define annihilation and creation (with dagger) operators

$$\begin{aligned}
\alpha &= \frac{1}{\sqrt{2}}\left(\psi^0 + i\psi^3\right), & \bar{\alpha}^\dagger &= \frac{1}{\sqrt{2}}\left(\overline{\psi}^0 - i\overline{\psi}^3\right) \\
\bar{\alpha} &= \frac{1}{\sqrt{2}}\left(\overline{\psi}^0 + i\overline{\psi}^3\right), & \alpha^\dagger &= \frac{1}{\sqrt{2}}\left(\psi^0 - i\psi^3\right)
\end{aligned} \tag{1.6}$$

and similarly for $m = 1, 2$ Lorentz components of the ψ we define β, $\bar{\beta}$, $\beta^\dagger$, $\bar{\beta}^\dagger$. The anti-commutator of a creation and an annihilation operator having inverse $\mathrm{U}_f(1)$ charges is 1. All the other anti-commutators vanish. In this basis

$$T = \alpha^\dagger\bar{\alpha} + \bar{\alpha}^\dagger\alpha + \beta^\dagger\bar{\beta} + \bar{\beta}^\dagger\beta \tag{1.7}$$

$Q^2 = 0$ implies that the space of states follow the pattern of the differential forms. Rank of the form is given by $2 - T$. Explicitly, the constrained $Q|\mathrm{phys}\rangle = 0$ is solved by $|\mathrm{phys}\rangle = Q|\mathrm{photon}\rangle$. Because $T|\mathrm{phys}\rangle = 0$ then $T|\mathrm{photon}\rangle = 1$. There are only four such states which constitute four components of the photon

$$|\mathrm{photon}\rangle = \{\bar{\alpha}^\dagger|0\rangle, \bar{\beta}^\dagger|0\rangle, \alpha^\dagger\bar{\alpha}^\dagger\bar{\beta}^\dagger|0\rangle, \bar{\alpha}^\dagger\beta^\dagger\bar{\beta}^\dagger|0\rangle\}\,, \tag{1.8}$$

which serve to build the photon field

$$A(x) = \left(A_+(x)\alpha^\dagger\bar{\alpha}^\dagger\bar{\beta}^\dagger + A_-(x)\bar{\beta}^\dagger - A_1(x)\bar{\alpha}^\dagger - A_2(x)\bar{\alpha}^\dagger\beta^\dagger\bar{\beta}^\dagger\right)|0\rangle \tag{1.9}$$

The gauge freedom is $|\mathrm{photon}\rangle \to |\mathrm{photon}\rangle + Q|\mathrm{sth}\rangle$. Because $T|\mathrm{sth}\rangle = 2$ the state $|\mathrm{sth}\rangle$ is uniquely given by $|\mathrm{sth}\rangle = \bar{\alpha}^\dagger\bar{\beta}^\dagger|0\rangle$.

1.1 Light-cone version

As we are going to propose a light-cone version of non-abelian gauge fields in the last section of this paper, here we describe shortly such a version for the above abelian theory. At the field theory level we set $A_+ = 0$. Then Maxwell equations yield a constraint $2\partial_+ A_- + \partial^i A_i = 0$ which can be solved for A_-. The perpendicular degrees of freedom (physical polarizations) are free fields. In the first quantized version this corresponds to gauging away $\alpha^\dagger \bar{\alpha}^\dagger \bar{\beta}^\dagger |0\rangle$. Then the physical modes are $\bar{\alpha}^\dagger|0\rangle$, $\bar{\alpha}^\dagger \beta^\dagger \bar{\beta}^\dagger |0\rangle$. They both are created from the light-cone "vacuum" $|0'\rangle = \bar{\alpha}^\dagger|0\rangle$ by creation operators $\beta^\dagger$, $\bar{\beta}^\dagger$ corresponding to the perpendicular modes. In terms of the path integral this means that we can choose the light-cone gauge fixing the local supersymmetry (1.2) by

$$\psi^+(\tau) = \overline{\psi}^+(\tau) = 0\,, \tag{1.10}$$

where $\psi^+ = \psi^0 + \psi^3$. Thus we are left only with two creation operators corresponding to the above $\beta^\dagger$, $\bar{\beta}^\dagger$. The T charge (1.7) also gets contribution from the β-modes only and its value on the light-cone vacuum is renormalized: $T|0\rangle = 0$. The constraint $T|\mathrm{phys}\rangle = 0$ leaves out two physical modes of the photon $|0\rangle$, $\beta^\dagger \bar{\beta}^\dagger |0\rangle$. The hamiltonian constraint (1.4) sets the equation of motion for the physical states.

This considerations will be necessary to understand the construction of a matrix model for YM_4 in section 3

2. Time independent matrix models

The purpose of this section is to generalize the first quantized version of a single gauge particle to the multi-particle case. Here we discuss extremally simplified models of time independent matrices. Thus we keep global $\mathcal{N} = 2$ SUSY as the only remnant of the local SUSY symmetry (1.2) of the spin-1 particle theory.

We introduce matter fields (X, ψ, F) as hermitean $N \times N$ matrices and put them into one $\mathcal{N} = 2$ supermultiplet

$$\hat{X} = X + i(\theta\overline{\psi} + \overline{\theta}\psi) + \theta\overline{\theta}F\,. \tag{2.1}$$

We also impose a $\mathrm{SU}(N)$ local gauge symmetry on the world-line under which the matter fields (X, ψ, F) will transform in the adjoint representation of $\mathrm{SU}(N)$. We want to to preserve shift symmetry $X \to X + a\mathbb{1}$. This forces the non-derivative interaction terms[2] to be dependent on commutators of $\hat{X}$. At the lowest order there is a unique such an operator which exists in arbitrary number of the target space dimensions:

$$\begin{aligned}\mathrm{tr}([\hat{X}^m, \hat{X}^n]^2)|_{\theta\overline{\theta}} = 4\,\mathrm{tr}\Big(&[X^m, X^n][X^m, F^n] - [X^m, X^n]\{\psi^m, \overline{\psi}^n\} - \\ &- [X^m, \psi^n][X^m, \overline{\psi}^n] - [X^m, \psi^n][\overline{\psi}^m, X^n]\Big)\end{aligned} \tag{2.2}$$

[2]Interaction terms with derivatives will not be discussed here.

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In various specific dimensions we have more possibilities. For 3d space we could have $\epsilon_{mnr}\,\mathrm{tr}([\hat{X}^m,\hat{X}^n]\hat{X}^r)|_{\theta\bar{\theta}}$ while 4d spaces allow: $\int d\tau\,\mathrm{tr}([\hat{X}^m,\hat{X}^n]*[\hat{X}^m,\hat{X}^n])|_{\theta\bar{\theta}}$ where $*$ denotes the Hodge star in the target space. If one extends the gauge multiplet then there are more choices. We are going to use this possibility in section 2.2. One can also add to the action the unity operator $\mathrm{tr}(\mathbb{1}) = N$. It looks trivial but can play an important role in the limit of infinite matrices.

All time independent $\mathcal{N} = 2$ matrix models posses a topological symmetry. The symmetry holds at the formal level and we shall not discuss its quantum fate e.g. possible anomalies. As all matrices are 1d time independent we suppress also the 1d gauge fields from the algebra. Now $\{Q_i, Q_j\} = 0$, where Q_i, $(i, j = 1, 2)$ are real components of Q. The SUSY transformations (1.2) are

$$\begin{aligned} \delta X &= i(\epsilon_1\psi_1 + \epsilon_2\psi_2) \\ \delta\psi_1 &= -\epsilon_2 F \\ \delta\psi_2 &= \epsilon_1 F \\ \delta F &= 0\,. \end{aligned} \tag{2.3}$$

It is clear that we have two candidates for BRST charges Q_i ($i = 1, 2$). They also respect $\{Q_1, Q_2\} = 0$. We choose Q_1 to be our BRST charge. Then ψ_1 is the ghost of the topological symmetry $\delta X =$ arbitrary matrix. Hence (ψ_2, F) form the anti-ghost system.

In the course of the paper we shall use these observations to show that the topological rigid string [8] has a natural matrix counterpart.

2.1 Large-N limits

In this subsection we shall describe a way one can take the limit of infinite rank of the gauge group. According to [14] the local $\mathrm{SU}(N)$ symmetry goes to $\mathrm{SDiff}(\Sigma_h)$ in the $N \to \infty$ limit. Moreover,

$$[A, B] \to P^{ab}\partial_a A\partial_b B\,, \qquad a = 1, 2\,, \tag{2.4}$$

where P is a Poisson structure on Σ_h and we substitute matrices by functions on a 2d compact parameter space (a Riemann surface Σ_h of genus h). If one equips Σ_h with a metric then one has a natural Poisson structure $P^{ab} = \epsilon^{ab}\partial_a A\partial_b B/\sqrt{g}$, where g is the determinant of a metric on Σ_h. Hence we face the problem how to choose the metric. In this paper we have decided to choose $g_{ab} = \partial_a X\partial_b X$ i.e. the induced metric. In this way we force the r.h.s. of (2.4) to be explicitly X dependent no matter what A and B are. The choice has several virtues which become apparent during the course of this article. In order to rewrite action for matrices we also substitute $\mathrm{tr}(\ldots) \to \int_{\Sigma_h}\sqrt{g}(\ldots)$.

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2.2 Relations topological strings

Let us consider the action

$$S = \{Q_1, V\}\,, \qquad V = \mathrm{tr}\,(\psi_2^m([X^n,[X^n,X^m]] + aF^m))\,. \tag{2.5}$$

For finite matrices the first term of this action is just (2.2). The model (2.5) is localized on matrices respecting[3]

$$[X^n,[X^n,X^m]] = 0\,. \tag{2.6}$$

After the substitution (2.4) the localization equations (2.6) go to

$$\Delta_g X^m = 0\,. \tag{2.7}$$

The laplacean Δ_g is defined with the 2d induced metric g_{ab}. With this prescription (2.5) defines the topological rigid string [8] in a flat space-time. To obtain the precise agreement with [8] we must sum over all possible genera h. This will find its independent confirmation in the next section. The formal equivalence of (2.5) with the topological rigid string is a striking result as the latter was claimed to be equivalent to the large-N limit of Yang-Mills theory (at least for some 2d target spaces). Hence we see that the gauge particle matrix model reproduce some feature of gauge fields. Moreover, in analogy to [8] we have

$$V = \left[Q_2, \mathrm{tr}\Big(-\frac{i}{4}[X^n,X^m]^2 + a\,\psi_2^m\psi_1^m\Big)\right] \tag{2.8}$$

what is a simple consequence of the $\mathcal{N}=2$ SUSY.

For 2d space-times the moduli space of (2.6) can be given more explicitly. Simple calculations (e.g. with help of the Cartan-Weyl basis) show that (2.6) is equivalent to

$$[X^1,X^2] = 0 \tag{2.9}$$

for finite matrices. For infinite matrices we can have [14]

$$[X^1,X^2] \sim 1\,. \tag{2.10}$$

We shall use both results in building relations with topological YM_2 and its $N\to\infty$ string picture of [7]. In order to achieve the goal we must slightly modify the previous construction. We extend the gauge multiplet to a full $\mathcal{N}=2$ multiplet as in [9]. Thus we take now

$$\begin{aligned}\delta\lambda &= i(\epsilon_1\eta_1 + \epsilon_2\eta_2)\\ \delta\eta_1 &= -\epsilon_2 A\\ \delta\eta_2 &= \epsilon_1 A\\ \delta A &= 0\end{aligned} \tag{2.11}$$

[3]One should not expect the localization to hold if the space of matrices is not compact in a way. The phenomenon might be similar to that which one finds for topological quantum mechanics on non-compact target spaces.

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and we choose the gauge fermion

$$V = \mathrm{tr}\left(\eta_2(\epsilon_{mn}[X^m, X^n] + bA) + [X^m, \lambda]\psi_1^m\right). \tag{2.12}$$

For finite N the model is localized on solutions to (2.9). On the other hand the $N \to \infty$ limit (2.4) of (2.12) is

$$V_\pm = \int_{\Sigma_h} \eta_2 \left[\epsilon_{mn}\epsilon^{ab}\partial_a X^m \partial_b X^n + bA\sqrt{g}\right] + \epsilon^{ab}\partial_a X^m \partial_b \lambda \psi_1^m . \tag{2.13}$$

The above is localized on both pseudo-holomorphic curves.[4]

$$\epsilon^{ab}\partial_a X^m \partial_b X^n = \pm\epsilon^{mn}\sqrt{g}. \tag{2.15}$$

The reason is that t^{mn} defined by $t^{mn} = \epsilon^{ab}\partial_a X^m \partial_b X^n/\sqrt{g}$ respects $|t^{mn}| = 1$. We also have $\Delta_g X^m = 0 \Leftrightarrow t^{mn} = \pm\epsilon^{mn}$. The equivalence holds for a compactification of both spaces of maps [7, 8]. We conclude that the theories (2.5) and (2.12) are equivalent for 2d target spaces. The result should hold for finite N and in the $N \to \infty$ limit.

2.3 Relation to topological YM$_2$

Here we consider compact target spaces of the theory (2.12). In general, the problem is not easy as we know from the standard (M)atrix model. The compactification on $M^2 = T^2$ is the best known example [2, 16]. For the SU(N) gauge group the relevant formula is

$$X^\mu \to D_\mu , \tag{2.16}$$

where D_μ is the covariant derivative. When applied to (2.12) it gives the standard 2d topological YM$_2$ theory [9],

$$V = \int_{T^2} \left(\phi(\epsilon^{mn}F_{mn} + bA) + (D_m\lambda)\psi_1^m\right). \tag{2.17}$$

We recognize that (2.17) is topological YM$_2$ localized on flat connections $F_{mn} = 0$ what corresponds to (2.9). It would be interesting to show that a similar correspondence holds for other target spaces e.g. the Riemann surfaces $M^2 = \Sigma_G$ of genus G. We do not have the proof of this fact but we notice that in general the topological theories are insensitive to many changes e.g. very often they are insensitive to the metric chosen on the target manifold [6]. With this assumption at hand we are ready to discuss the $N \to \infty$ limit.

[4] Pseudo-holomorphic curves are defined to fulfill

$$\frac{\epsilon_a^{\ b}}{\sqrt{g}}\partial_b X^m \pm J_n^{\ m}\partial_a X^n = 0 , \tag{2.14}$$

where $J_n^{\ m}$ is a 2d complex structure defined on M^2 [15].

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2.4 Relation between 2d topological string models and topological YM_2

All matrix topological theories with 2d target spaces described in the previous subsections define the same moduli problem $[X^m, X^n] = 0$. As we claimed previously (see (2.16)) for the space-time $M^2 = T^2$ this appears to the same as $F_{mn} = 0$ i.e. the theory of flat $\mathrm{SU}(N)$ connections [9]. On the other hand if we first take the limit $N \to \infty$ we get the topological rigid string or equivalently, the theory of pseudo-holomorphic maps $t^{mn} = \pm 1$ from a world-sheet Riemann surface Σ_h to the 2d target manifold. This shows that there should be a relation between both theories. In fact, the relation is well known [17, 7, 8]: both theories are equivalent in the $1/N$ expansion.[5]

$$Z_{\mathrm{YM}_2}(\mathrm{SU}(N), G) = \sum_h \left(\frac{1}{N}\right)^{2h-2} Z_{\text{topological strings}}(X : \Sigma_h \to \Sigma_G) \tag{2.18}$$

The above formula shows explicitly the meaning to the genus h which appeared when we have taken the limit $N \to \infty$ (2.4).

3. Quantum mechanical matrix models with gauge symmetry.

Although the discussed topological models left several problems unsolved we feel obliged to say something about the next motivation of this work i.e. the possible application to gauge fields dynamics in four dimensions. The matrix model described in this section goes beyond time independent matrix models. In fact it is a quantum mechanical matrix model based on previous discussed global $\mathcal{N} = 2$ supersymmetry of the gauge particle. As discussed before we are bound to use non-covariant formulation in order to describe multi-particle quantum mechanics. Thus we should interpret the theory as the parton model of gauge particles in the DLCQ limit [12]. The model breaks explicitly the Lorentz group for finite N but the hope is that after taking the $N \to \infty$ limit the symmetry will be recovered. We stick to 4d space-time so the light-cone target space X^m is 2-dimensional (m, $n = 1, 2$) and the superfield $\hat{X}^m$ is a hermitean $N \times N$ matrix.

We want also that the model possesses local $\mathrm{U}_f(1) \times \mathrm{SU}(N)$ symmetry thus we need gauge fields f, A. Now the SUSY transformations are

$$\begin{aligned} \delta X &= i\epsilon\overline{\psi} + \text{h.c.} \\ \delta\psi &= \epsilon(-DX + ieF) \\ \delta F &= \epsilon D\overline{\psi} + \text{h.c.} \end{aligned} \tag{3.1}$$

[5] This has been shown for $M^2 = \Sigma_G$ of genus at least 2 [7]. For $G = 1$ one needs the cosmological (Nambu-Goto) term in order the partition function does not vanish. For $G = 0$ the situation is more complicated [18].

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All derivatives in the above are defined with the $\mathrm{U}_f(1) \times \mathrm{SU}(N)$ connection A i.e. $DX \equiv (\partial_\tau X - i[A,X])$ and $D\psi \equiv (\partial_\tau \psi - if\psi - i[A,\psi])$, $D\overline{\psi} \equiv (\partial_\tau \overline{\psi} + if\overline{\psi} - i[A,\overline{\psi}])$.

The kinetic term is given by

$$\begin{aligned} L_0 &= \frac{1}{2}\,\mathrm{tr}(|D_\theta \hat{X}|^2)|_{\theta\overline{\theta}} \\ &= \frac{1}{2}\,\mathrm{tr}\left((DX)^2 + i\psi D\overline{\psi} + i\overline{\psi} D\psi + F^2\right) \end{aligned} \tag{3.2}$$

where $D_\theta \hat{X} \equiv \partial_\theta \hat{X} - i\overline{\theta} D\hat{X}$. The covariant derivative is understood as in (3.1).

Thus the proposed action is

$$L = \left.\left(\frac{1}{2}|D_\theta \hat{X}^m|^2 + \lambda\,\mathrm{tr}\left([\hat{X}^m, \hat{X}^n]^2\right)\right)\right|_{\theta\overline{\theta}}, \tag{3.3}$$

where λ is a dimension 2 coupling constant. We analyze first the constraints which follow from the gauge invariance $\mathrm{U}_f(1) \times \mathrm{SU}(N)$. They are $T|\mathrm{phys}\rangle = T^a|\mathrm{phys}\rangle = 0$ $(a = 1, \ldots, N^2 - 1)$. We notice also that the constraints do not depend on λ.

Defining creation and annihilation operators as in (1.7) we get operators the following normal ordered form of T and T^a:

$$T = \beta^{a\dagger}\bar{\beta}^a + \bar{\beta}^{a\dagger}\beta^a, \quad T^a = -if^{abc}(\beta^{b\dagger}\bar{\beta}^c + \bar{\beta}^{b\dagger}\beta^c + iX^{bm}\Pi^{cm}) \tag{3.4}$$

The full analysis of (3.3) goes beyond the scope of this letter and will be the subject of a future publication. Here we limit its discussion to few comments. The expression defined by (3.3) should be interpreted as DLCQ hamiltonian. Some of its eigenvalues are apparent: these are the degrees of freedom describing the motion of the center of mass of the system $X^m \propto 1$. They do not interact with the other degrees of freedom. The structure of the constraints is analogous to what was obtained for the pure QED case in section 1. The hamiltonian constraint (1.4) of this section is just the evolution equation here. Also for the light-cone gauge we do not need supersymmetry constraints (1.3). The only non-trivial constraints are $T|\mathrm{phys}\rangle = 0 = T^a|\mathrm{phys}\rangle$ which looks like a natural extension of (1.5). Thus we may hope to deal with the physical states only.

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References

[1] M.B. Halpern and C. Schwartz, *The algebras of large-N matrix mechanics*, *Int. J. Mod. Phys.* **A 14** (1999) 3059 [hep-th/9809197];
M. Staudacher, *large-N matrix field theories*, *Fortsch. Phys.* **48** (2000) 217 [hep-th/9904111].

[2] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, *M-theory as a matrix model: a conjecture*, *Phys. Rev.* **D 55** (1997) 5112 [hep-th/9610043];
N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, *A large-N reduced model as superstring*, *Nucl. Phys.* **B 498** (1997) 467 [hep-th/9612115];
D. Bigatti and L. Susskind, *Review of matrix theory*, hep-th/9712072.

[3] B. de Wit, J. Hoppe and H. Nicolai, *On the quantum mechanics of supermembranes*, *Nucl. Phys.* **B 305** (1988) 545;
B. de Wit, *Supermembranes and super matrix models*, hep-th/9902051.

[4] G. Gabadadze and Z. Kakushadze, *Monopole condensation, matrix quantum mechanics and membrane approach in QCD*, hep-th/9905198; *A remark on witten effect for QCD monopoles in matrix quantum mechanics*, *Mod. Phys. Lett.* **A 14** (1999) 2151 [hep-th/9908039];
M. Billò, M. Caselle, A. D'Adda and P. Provero, *2d Yang-Mills theory as a matrix string theory*, hep-th/9901053.

[5] J. Plefka, M. Serone and A. Waldron, *Matrix theory and feynman diagrams*, *Fortsch. Phys.* **48** (2000) 191 [hep-th/9903099].

[6] E. Witten, *Topological quantum field theory*, *Comm. Math. Phys.* **117** (1988) 353;
D.Birmingham, M.Blau, M.Rakowski, G.Thompson, *Topological field theory*, *Phys. Rep.* **209** (1991) 129.

[7] S. Cordes, G. Moore and S. Ramgoolam, *large-N 2-d Yang-Mills theory and topological string theory*, *Comm. Math. Phys.* **185** (1997) 543 [hep-th/9402107].

[8] P. Hořava, *Topological strings and QCD in two-dimensions*, hep-th/9311156; *Topological rigid string theory and two-dimensional QCD*, *Nucl. Phys.* **B 463** (1996) 238 [hep-th/9507060];
A.M.Polyakov, Lectures given at the CRM-CAP Summer School *Particle and Fields '94*, August 16-24 1994, Banff, Alberta, Canada.

[9] E.Witten, *Two-dimensional gauge theories revisited*, *J. Geom. Phys.* **9** (1992) 303 [hep-th/9204083].

[10] P.Howe, S.Penati, M.Pernici and P.Townsend, *Wave equations for arbitrary spin from quantization of the extended supersymmetric spinning particle*, *Phys. Lett.* **B 215** (1988) 555.

[11] L. Susskind, *Another conjecture about m(atrix) theory*, hep-th/9704080.

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[12] J.Kogut and L. Susskind, *The Parton picture of elementary particles*, *Phys. Rep.* **8** (1973) 75;
D. Bigatti and L. Susskind, *Review of matrix theory*, `hep-th/9712072`.

[13] *Theory of hadrons and light front QCD.* Proceedings, 4th International workshop on light front quantization and nonperturbative dynamics, Polana Zgorzelisko, S.D. Glazek ed., World Scientific, 1995.

[14] J.Hoppe, PhD thesis, MIT 1982;
E.G. Floratos, J. Iliopoulos and G. Tiktopoulos, *A note on* SU(∞) *classical Yang-Mills theory*, *Phys. Lett.* **B 217** (1989) 285;
D.B. Fairlie and C.K. Zachos, *Infinite dimensional algebras, sine brackets and* SU(∞), *Phys. Lett.* **B 224** (1989) 101;
C.N. Pope and K.S. Stelle, SU(∞), SU$^+$(∞) *and area preserving algebras*, *Phys. Lett.* **B 226** (1989) 257.

[15] J. Pawełczyk, *Rigid string instantons are pseudo-holomorphic curves*, *Nucl. Phys.* **B 491** (1997) 515 [`hep-th/9609196`].

[16] A. Connes, M.R. Douglas and A. Schwarz, *Noncommutative geometry and matrix theory: compactification on tori*, *J. High Energy Phys.* **02** (1998) 003 [`hep-th/9711162`];
H. Aoki, N. Ishibashi, S. Iso, H. Kawai, Y. Kitazawa, T. Tada, *Noncommutative Yang-Mills in IIB matrix model*, `hep-th/9908141`;
E. G. Floratos, G. K. Leontaris, *The Matrix model and the non-commutative geometry of the supermembrane*, *Phys. Lett.* **B 464** (1999) 30 [`hep-th/9908106`].

[17] D.J. Gross, *Two-dimensional QCD as a string theory*, *Nucl. Phys.* **B 400** (1993) 161 [`hep-th/9212149`];
D.J. Gross and I.W. Taylor, *Two-dimensional QCD is a string theory*, *Nucl. Phys.* **B 400** (1993) 181 [`hep-th/9301068`]; *Twists and wilson loops in the string theory of two- dimensional QCD*, *Nucl. Phys.* **B 403** (1993) 395 [`hep-th/9303046`].

[18] M.R. Douglas and V.A. Kazakov, *large-N phase transition in continuum QCD in two-dimensions*, *Phys. Lett.* **B 319** (1993) 219 [`hep-th/9305047`];
D.J. Gross and A. Matytsin, *Instanton induced large-N phase transitions in two- dimensional and four-dimensional QCD*, *Nucl. Phys.* **B 429** (1994) 50 [`hep-th/9404004`].